SMI-9-94

Generalized Pauli-Villars regularization for undoubled lattice fermions

A.A.Slavnov[1]
Steklov Mathematical Institute, Russian Academy of Sciences,
Vavilov st.42, GSP-1,117966, Moscow, Russia

Abstract

A manifestly gauge invariant formulation of chiral theories with fermions on the lattice is developed. It combines SLAC lattice derivative [1], [2], [3] and generalized Pauli-Villars regularization [4]. The theory is free of fermion doubling, requires only local gauge invariant counterterms and produces correct results when applied to exactly solvable two dimensional models.

1 Introduction

In this paper I discuss a possibility to describe chiral fermions on the lattice in the framework of the SLAC formulation [1], [2], [3] supplemented by the additional Pauli-Villars (PV) type regularization which allows to avoid the problems of the original SLAC model.

It is known that due to nonlocality of the SLAC action perturbation theory suffers from the appearance of nonlocal counterterms and spurious infrared divergencies [5]. The attempts to overcome this difficulties by making a partial resummation [6] when applied to solvable models lead to the wrong physical spectrum [7].

The origin of these difficulties is a singular behavior of the fermion propagator and vertices near the edge of the Brillouin zone $p_\mu \sim \frac{\pi}{a}$. The contribution of this region produces nonlocal counterterms and infrared divergencies. Essentially the same problem is present in all other models proposed to eliminate fermion doubling on the lattice [8], [9], [10]. (For detailed references see [11], [12], [13]). In particular in Wilson formulation the contribution of the region near the edge of the Brillouin zone produces gauge noninvariant counterterms. In other words a lattice formulation is not a bona fide regularization for chiral fermions, and to get an invariant calculational scheme one needs to suppress a contribution of momenta of the order of cut-off. In fact it is the same problem which has to be solved if one wishes to

[1]E-mail: slavnov@qft.mian.su

construct a gauge invariant regularization for the chiral fermions in the continuum theory.

Recently we proposed a manifestly invariant regularizaion for the anomaly free chiral models, in particular for the standard model [4] (see also [14]). Our procedure is a generalization of a Pauli-Villars regularization which allows for an infinite number of auxiliary PV fields. It has been shown that when applied to the chiral models on the lattice this procedure leads to a gauge invariant continuum theory without fermion doubling both in the case of Wilson fermions [15] and in the Smit-Swift model [16]. However both these formulations have some drawbacks. It is a lack of a gauge invariance for a finite lattice spacing in the first case, and the necessity to introduce an additional Yukawa interaction in the second case. It seems that the most appropriate way to implement this regulaization in lattice models is to use the SLAC formulation.

We shall show that if one writes a lattice regularization for anomaly free chiral gauge models following [1] [2], [3] and introduces simultaneously PV fields according to [4],the resulting model is manifestly gauge invariant, has no fermion doubling, and does not require nonlocal counterterms in perturbation theory. When applied to solvable two dimensional models this procedure leads to a correct continuum limit.

2 Four dimensional $SO(10)$ model.

In this section we consider the grand unified $SO(10)$ model. It is worthwhile to emphasize that representations we consider are not real, and the choice of $SO(10)$ is explained by the possibility to write all the formulae in a compact form. We could consider equally well the standard model or Weinberg-Salam model provided the lepton and quark representations are choosen in such a way that anomalies are compensated. The SLAC lattice action for the grand unified $SO(10)$ model looks as follows:

$$I = \sum_{k,\mu,x,y} \bar{\psi}_+^k(x)\gamma_\mu i D_\mu(x-y) P \exp\{i \sum_{z_\mu = x_\mu}^{y_\mu} g A_\mu(z)\} \psi_+^k(y) \tag{1}$$

Here $A_\mu = A_\mu^{ij}\sigma_{ij}$. The matrices σ_{ij} are the $SO(10)$ generators:$\sigma_{ij} = 1/2[\Gamma_i, \Gamma_j]$, where Γ_i are Hermitian 32×32 matrices which satisfy the Clifford algebra: $[\Gamma_i, \Gamma_j] = 2\delta_{ij}$. The chiral $SO(10)$ spinors $\psi_\pm = 1/2(1 \pm \Gamma_{11})\psi$, where $\Gamma_{11} = \Gamma_1\Gamma_2 \ldots \Gamma_{10}$, describe the 16-dimensional irreducible representation of $SO(10)$ including quark and lepton fields. We assume also that the spinors $\psi_\pm$ are left-handed $\psi_\pm = 1/2(1 + \gamma_5)\psi_\pm$. Index k numerates different generations. $D_\mu(x)$ is the SLAC derivative

$$D_\mu(x) = \int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^4k}{(2\pi)^4} ik_\mu \exp\{ikx\} \tag{2}$$

The action (1) is manifestly invariant with respect to $SO(10)$ gauge transformations and does not suffer from fermion doubling. However it is not local which leads to difficulties we mentioned above.

To avoid these problems one modifies the action (1) by adding the analogous interaction of PV fields [4], [15], [16]

$$I \to I + I_{PV} \tag{3}$$

$$I_{PV} = \sum_{r,\mu,x,y} \bar{\psi}_r(x)\gamma_\mu i D_\mu(x-y) P\exp\{i\sum_{z_\mu=x_\mu}^{y_\mu} gA_\mu(z)\}\psi_r(y) - \frac{M_r}{2}\bar{\psi}_r(x)C_D C\Gamma_{11}\bar{\psi}_r^T +$$

$$+\sum_{r,\mu,x,y} \bar{\Phi}_r(x)\gamma_\mu i\Gamma_{11} D_\mu(x-y) P\exp\{i\sum_{z_\mu=x_\mu}^{y_\mu} gA_\mu(z)\}\Phi_r(y) - \frac{M_r}{2}\bar{\Phi}_r(x)C_D C\bar{\Phi}_r^T + h.c.$$

Here ψ_r are anticommuting PV fields and Φ_r are commuting PV fields, which realize the reducible 32 dimensional representation of $SO(10)$, C_D is the usual charge conjugation matrix and C is the $SO(10)$ conjugation matrix $\sigma_{ij}^T C = -C\sigma_{ij}$.

The action (3) is manifestly invariant with respect to the $SO(10)$ gauge transformations. Now we shall show that it generates a perturbation theory which in the limit $a \to 0$ results in a continuum theory with only local gauge invariant counterterms. Nonlocal counterterms do not appear.

The perturbative propagators look as follows

$$S_{\bar{\psi}^+\psi^+} = \frac{\hat{P}}{P^2} \tag{4}$$

$$S_{\bar{\psi}_r^+\psi_r^+} = S_{\bar{\psi}_r^-\psi_r^-} = S_{\bar{\Phi}_r^+\Phi_r^+} = -S_{\bar{\Phi}_r^-\Phi_r^-} = \frac{\hat{P}}{P^2+M_r^2}, \tag{5}$$

$$S_{\bar{\psi}_r^-\bar{\psi}_r^+} = S_{\psi_r^+\psi_r^-} = S_{\bar{\Phi}_r^-\bar{\Phi}_r^+} = S_{\Phi_r^+\Phi_r^-} = \frac{M_r C_D C\Gamma_{11}}{P^2+M_r^2}, \tag{6}$$

where P_μ is the sawtooth function

$$P_\mu(p) = p_\mu - 2m\frac{\pi}{a}; \quad (2m-1)\frac{\pi}{a} < p_\mu < (2m+1)\frac{\pi}{a}. \tag{7}$$

The expansion of the action (3) in terms of g generates the interaction vertices with the increasing number of vector lines. (For detailes see [5], [6]). The three point vertex looks as follows

$$\Gamma^3 = g\gamma_\mu \frac{P_\mu(p) - P_\mu(q)}{K_\mu(k)}\sigma_{ij}, \tag{8}$$

with

$$K_\mu(k) = \frac{\exp[iak_\mu - 1]}{ia} \tag{9}$$

One sees that the interaction vertex (8) is nonlocal, and it introduces additional singularities into Feynman diagrams. Indeed, if $p < \frac{\pi}{a}$, but $q > \frac{\pi}{a}$, the interaction vertex becomes $\sim \frac{a^{-1}}{K_\mu(k)}$, which results in the appearance of the nonlocal divergent contributions [5].

When the PV fields are introduced the contribution of the region $p \sim a^{-1}$ is suppressed and nonlocal divergencies do not appear.

Let us consider for example the vacuum polarisation diagrams generated by the vertex (8). They look as follows:

$$\Pi^{(\pm)r}_{\mu\nu} = -\frac{g^2}{K_\mu K_\nu}\int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^4 l}{(2\pi)^4}\,\mathrm{Tr}\,[\,\sigma_{ij}(1\pm\Gamma_{11})\sigma_{kl}]\times \tag{10}$$

$$\mathrm{Tr}\,[\,(1+\gamma_5)\gamma_\mu(\hat{P}(l)+M_r)\gamma_\nu(\hat{P}(l+p)+M_r)]\frac{[P_\mu(l)-P_\mu(l+p)][P_\nu(l+p)-P_\nu(l)]}{[P^2(l)+M_r^2][P^2(l+p)+M_r^2]}.$$

(We consider the contribution of the chirality preserving propagators (4, 5)). The contribution of the original fermion is given by $\Pi^{+(0)}_{\mu\nu}$, $(M_r=0)$. Contribution of the bosonic fields differs by sign.

First of all we note that

$$\mathrm{Tr}\,[\,\sigma_{ij}\Gamma_{11}\sigma_{kl}]=0. \tag{11}$$

and therefore positive and negative "chirality" PV fields give the same contribution to $\Pi_{\mu\nu}$. Secondly, the nonlocal contribution comes from the region of integration where $\frac{\pi}{a}-|p_\mu|\le|l_\mu|\le\frac{\pi}{a}$. If one chooses $M_r\ll a^{-1}$, one can expand the integrand in this region in terms of M_r. Terms of zero order produce the total contribution proportional to

$$k+2\sum_r(-1)^r c_r \tag{12}$$

Here c_r is the number of the PV fields with the mass M_r. The contributions of fermionic and bosonic fields differ by sign. The factor 2 arises because there are PV fields of both chiralities, and the factor k is due to the presence of k generations of original fermions. If the number of generations is even one can always choose the coefficients c_r in such a way that

$$k+2\sum_r(-1)^r c_r=0 \tag{13}$$

Imposing the further PV conditions

$$\sum_r c_r(-1)^r M_r^2=0 \tag{14}$$

one can make the contribution of the domain $\frac{\pi}{a}-|p_\mu|\le|l_\mu|\le\frac{\pi}{a}$ vanishing in the limit $a\to0$. In the remaining integral over $|l_\mu|\le\frac{\pi}{a}-|p_\mu|$,

$$P_\mu(l)-P_\mu(l+p)\sim p_\mu \tag{15}$$

Therefore the nonlocal factors $K_\mu(p)$ are compensated and in the continuum limit one gets a manifestly gauge invariant expression for the polarization operator.

The same arguments are obviously applicable to the diagrams including chirality changing propagators (6).

In the case of an odd number of generations the eq.(13) cannot be satisfied for any finite number of PV fields, as by construction the coefficients c_r are integer. As was discussed in ref. [4], [15], [16] in this case the desired suppression of the region near the edge of the Brillouin zone may be achieved if one introduces an infinite

series of PV fields with the masses $M_r = Mr$, $M(a) \to \infty$ when $a \to 0$, $M \ll a^{-1}$. In this case after summation over r one gets the integrand

$$\sim \frac{\pi}{MR\sinh(\pi RM^{-1})}, \quad R^2 = P_0^2(l) + P_1^2(l) + P_2^2(l) + P_3^2(l) \tag{16}$$

which vanishes exponentially for $R \sim \frac{\pi}{a}$. Therefore this region does not contribute and one can use the same arguments as above to show that in the continuum limit one gets a manifestly gauge invariant polarization operator and only usual local gauge invariant counterterms are needed.

The proof for the diagrams with 3 and 4 external lines is given in the same way. The diagrams generated by the vertex function (8) and the propagators (4),(5) may be presented in the form

$$I_{\mu_1\ldots\mu_n} = \frac{\tilde{I}_{\mu_1\ldots\mu_n}}{K_{\mu_1}(p_1)K_{\mu_2}(p_2)\ldots K_{\mu_n}(p_n)} \tag{17}$$

where

$$\tilde{I}^{(\pm)r}_{\mu_1\ldots\mu_n} = -g^n \int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^4l}{(2\pi)^4} \mathrm{Tr}\,[\,\sigma_{ij}(1 \pm \Gamma_{11})\sigma_{kl}\ldots\sigma_{mn}] \times \tag{18}$$

$$\mathrm{Tr}\,[\,(1+\gamma_5)\gamma_{\mu_1}(\hat{P}(l) + M_r)\gamma_{\mu_2}\ldots(\hat{P}(l-p_n) + M_r)]\times$$

$$\frac{[P_{\mu_1}(l) - P_{\mu_1}(l+p_1)]\ldots[P_{\mu_n}(l-p_n) - P_{\mu_n}(l)]}{[P^2(l) + M_r^2]\ldots[P^2(l-p_n) + M_r^2]}.$$

Due to the properties of Γ matrices $\mathrm{Tr}\,[\,\sigma_{i_1j_1}\Gamma_{11}\ldots\sigma_{i_nj_n}] = 0$ for $n \leq 4$ and as before the contributions of positive and negative chirality fermions are equal. The integrals over dl_i may be separated into two parts:

$$\int_{V_{in}} dl_i + \int_{V_{out}} dl_i, \quad V_{in} = |l_i| < \frac{\pi}{a} - |p_i|, \quad V_{out} = |l_i| > \frac{\pi}{a} - |p_i| \tag{19}$$

Here $|p_i| = \sup|p_i^1 + \ldots p_i^l|, l = 1, \ldots n-1$. If l_μ belongs to V_{in} then the factors $P_\mu(l + p^1 + \ldots p^l) - P_\mu(l + p^1 + \ldots p^{l-1})$ can be replaced by p_μ^l,and the integral over V_{in} reduces in the limit $a \to 0$ to the usual gauge invariant continuum expression. Hence if we adjust the parameters of the PV fields in such a way that the integral over V_{out} vanishes in the continuum limit, our statement will be proven.

In complete analogy with the analysis of the polarization operator in the case of even number of generations of the original fields we may take a finite number of PV fields with the parameters satisfying eqs. (13, 14). Expanding the integrands in (18) one sees that the leading terms cancel and the remaining terms vanish in the continuum limit.

In the case of an odd number of generations one needs an infinite system of PV fields. Taking the traces and separating tensorial structures in eq. (18) one can write for $\tilde{I}_{\mu_1\ldots\mu_n}$

$$\tilde{I}_{\mu_1\ldots\mu_n} \sim \int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^4l}{(2\pi)^4} \sum_{r=-\infty}^{+\infty} \sum_{j=0}^{n-1} \frac{A_j^{\mu_1\ldots\mu_n}(l, Q, M_r)}{P^2(l+Q_j) + M^2r^2}, \quad Q_j = p_1 + \ldots + p_j. \tag{20}$$

where $A_l^{\mu_1 \dots \mu_n}$ is a polinomial in M_r^2. The summation over r can be done explicitly as in ref.[4], [15]. One gets

$$I_{\mu_1 \dots \mu_n} \sim \int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^4 l}{(2\pi)^4} \sum_{j=0}^{n-1} \frac{\tilde{A}_j^{\mu_1 \dots \mu_n}(l, Q,)}{MR \sinh(\pi R M^{-1})} \tag{21}$$

In the region V_{out} the integrand vanishes exponentially when $a \to 0$, and the integral over V_{in} reduces to the gauge invariant continuum expression.

To extend this proof to the higher order diagrams we firstly note that contrary to the statements which one can meet in the literature the model (1) generates a finite number of types of divergent diagrams. Indeed, scaling the integration variables $l_\mu \to l_\mu a^{-1}$, in eq.(18) one would get for arbitrary diagram the highest degree of divergency $\sim a^{-4}$. However this estimation is too rude. Let us show that all the diagrams with $n \geq 5$ are convergent. To prove it we separate the domain of integration overdl_i as in eq.(19). If l_μ belongs to V_{in} then the factors $P_\mu(l + p^1 + \dots p^l) - P_\mu(l + p^1 + \dots p^{l-1})$ can be replaced by p_μ^l. Each such factor decreases the degree of divergency by one. The integral over V_{out} is limited by the product of independent external momenta $p_1 p_2 \dots p_{n-1}$. Indeed, the integrand contains the factors

$$P_i(l + ap_1 + \dots + ap_k) - P_i(l + ap_1 + \dots + ap_{k+1}) \tag{22}$$

This factor can be different of ap_{k+1}^i only on the interval of values of l^i which is equal to ap_{k+1}^i. Noting that the remaining terms in the integrand are not singular at $p_{k+1}^i = 0$, we conclude that the integral is limited by $F|ap_{k+1}^i|$ where F is a polynomial over ap_{k+1}^i. It follows that the diagram with n external lines contributes the factor

$$\tilde{I}_{\mu_1 \dots \mu_n} \leq a^{-4} |ap_{\mu_1}| \dots |ap_{\mu_n - 1}| \tag{23}$$

and therefore all the diagrams with $n \geq 5$ are convergent.

These reasonings are easily extended to the case when the diagram includes the higher order vertices $\Gamma_n(l, p_1 \dots p_n)$. The vertices Γ_n are defined up to nonessential factors by the recurrent relations

$$\Gamma_{n+1}(l, p_1 \dots p_{n+1} \sim -\Gamma_n(l, p_1 \dots p_n) + \Gamma_n(l + p_{n+1}, p_1 \dots p_n) \tag{24}$$

Let us consider some momentum p_i^k.It follows from eqs.(8, 24) that Γ_n is the sum of P_i depending on different arguments and all the terms in Γ_n may be separated into pairs whose arguments differ by p_i^k. Therefore Γ_n is proportional to p_i^k everywhere except for the interval of the values of l_i whose length is equal to ap_i^k. In complete analogy with the case discussed above we conclude that the eq.(23) holds. Counting the number of independent momenta one sees that the diagrams with $n > 4$ are convergent. In the limit $a \to 0$ they coincide with the gauge invariant continuum expression. The corresponding diagrams with PV fields vanish in the continuum limit.

It is worthwhile to notice that these reasonings do not exclude nonlocal singularities for diagrams with $n < 4$. For example in polarization operator there is only

one independent momentum and without PV regularization it contains nonlocal singularities.

This discussion shows also that spurious infrared divergencies which could be caused by the singularities of the vertex functions are in fact absent being supressed according to the eq.(23).

The analysis of the diagrams with divergent subgraphs in particular overlapping divergencies requires more complicated machinery and has not yet been done. We just mention that this problem may be avoided by introducing an additional higher derivative regularization for Yang-Mills field, as it has been done in the refs. [15], [16]. With this regularization all the above results are trivially extended to arbitrary diagrams.

It completes the proof of the absence of nonlocal counterterms in our scheme. Of course all the arguments given above used the weak coupling expansion and it would be very important to verify them nonperturbatively. Having this in mind we consider in the next section two dimensional chiral models.

3 Two dimensional models

In two dimensional models the procedure described above simplifies greatly because the only divergent diagram is the second order polarization operator. In the continuum Abelian model the polarization operator is the only nonzero diagram and for that reason the anomaly free chiral models are essentially equivalent to the vectorial Schwinger model. It is not true in the nonabelian case.

We start with the anomaly free chiral Schwinger model. The fermionic part of the action is

$$I=\sum_{\mu,x,y}[\bar{\psi}_+(x)\gamma_\mu iD_\mu(x-y)\exp\{iea\sum_{z_\mu=x_\mu}^{y_\mu}A_\mu(z)\}\psi_+(y)+ \tag{25}$$

$$\sum_{k=1,2}\bar{\psi}^k_-(x)\gamma_\mu iD_\mu(x-y)\exp\{i\frac{e}{\sqrt{2}}a\sum_{z_\mu=x_\mu}^{y_\mu}A_\mu(z)\}\psi^k_-(y)]$$

where

$$D_\mu(x)=\int_{\frac{-\pi}{a}}^{\frac{\pi}{a}}\frac{d^2k}{(2\pi)^2}ik_\mu\exp\{ikx\} \tag{26}$$

$\psi_\pm=\frac{1}{2}(1\pm\gamma_5)\psi$. The coupling constants of the positive and negative chirality fermions are adjusted in such a way that the γ_5 contributions cancel and the model is anomaly free.

To suppress the contribution of the region near the edges of the Brillouin zone we introduce the vectorial interaction of bosonic PV fields Φ

$$I=\sum_{\mu,x,y}[\bar{\Phi}(x)\gamma_\mu iD_\mu(x-y)\exp\{iea\sum_{z_\mu=x_\mu}^{y_\mu}A_\mu(z)\}\Phi(y)+M\bar{\Phi}(x)\Phi(x)] \tag{27}$$

The regularized action $I_R = I + I_{PV}$ is invariant with respect to the gauge transformations

$$\psi_+(x) \to \exp\{iea\xi(x)\}\psi_+(x); \quad \psi^k_-(x) \to \exp\{i\frac{e}{\sqrt{2}}a\xi(x)\}\psi_-(x) \tag{28}$$

$$\Phi(x) \to \exp\{iea\xi(x)\}\Phi(x), A_\mu(x) \to A_\mu(x) + a^{-1}[f(x_\mu + a) - f(x)].$$

Consider firstly the vacuum polarization graph. The corresponding amplitudes for each fermions are

$$\Pi^{(\pm)a}_{\mu\nu} = -\frac{{g_\pm}^2}{K_\mu K_\nu}\int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^2 l}{(2\pi)^2}\,\mathrm{Tr}\,[\,\frac{1}{2}(1 \pm \gamma_5)\gamma_\mu \hat{P}(l)\gamma_\nu \hat{P}(l+p)] \times \tag{29}$$
$$\frac{[P_\mu(l) - P_\mu(l+p)][P_\nu(l+p) - P_\nu(l)]}{P^2(l)P^2(l+p)}.$$

$$\Pi^{(\pm)b}_{\mu\nu} = -\frac{g^2_\pm \delta_{\mu\nu}}{K_\mu K_\nu}\int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^2 l}{(2\pi)^2}\,\mathrm{Tr}\,[\,\frac{1}{2}(1 \pm \gamma_5)\gamma_\mu(\hat{P}(l)]\frac{[-2P_\mu(l) + P_\mu(l+p) + P_\mu(l-p)]}{P^2(l)} \tag{30}$$

Here $\pm$ stands for the contributions of positive and negative chirality fermions , and $g_+ = e, g_- = \frac{e}{\sqrt{2}}$. The corresponding amplitudes for the PV fields are

$$\Pi^a_{\mu\nu} = \frac{e^2}{K_\mu K_\nu}\int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^2 l}{(2\pi)^2}\,\mathrm{Tr}\,[\,\gamma_\mu(\hat{P}(l) + M_r)\gamma_\nu(\hat{P}(l+p) + M_r)] \times \tag{31}$$

$$\frac{[P_\mu(l) - P_\mu(l+p)][P_\nu(l+p) - P_\nu(l)]}{[P^2(l) + M_r^2][P^2(l+p) + M_r^2]}.$$

$$\Pi^b_{\mu\nu} = 2\frac{e^2\delta_{\mu\nu}}{K_\mu K_\nu}\int_{\frac{-\pi}{a}}^{\frac{\pi}{a}} \frac{d^2 l}{(2\pi)^2}\,\mathrm{Tr}\,[\,\gamma_\mu(\hat{P}(l) + M_r)]\frac{[-2P_\mu(l) + P_\mu(l+p) + P_\mu(l-p)]}{P^2(l) + {M_r}^2} \tag{32}$$

The complete contribution of the original fermions is given by

$$\Pi^{a+}_{\mu\nu} + 2\Pi^{a-}_{\mu\nu} + \Pi^{b+}_{\mu\nu} + 2\Pi^{b-}_{\mu\nu} \tag{33}$$

The terms proportional to γ_5 cancel and the sum (33) coincides up to the sign and mass terms with the PV amplitudes (31, 32). Following the procedure described in the preceeding section we can separate the integration domain into V_{in} and V_{out}. In the domain V_{out}, $p \gg M$ and one can expand the integrands in terms of M. The first terms cancel and the next terms are majorated by M^2a^2. Therefore the integral over V_{out} vanishes in the continuum limit. The integral over V_{in} in the limit $a \to 0$ coincides with the PV regularized continuum theory. The same reasoning are applied when l_0 belongs to V_{in} and l_1 belongs to V_{out} and visa versa.

The diagrams with more than two photon lines are analysed in the same way as above. One can easily see that the infrared divergencies are absent. Indeed, according to the estimate given above the integrals over fermion loops are limited by the product of independent external momenta. Therefore the infrared singular factors are supressed. A straightforward power counting shows that the higher order

sea-gull vertices give a contribution vanishing in the continuum limit. For example the tadpole diagram produced by the second order vertex contributes the factor

$$\Pi \sim \int_{-\frac{\pi}{a}}^{\frac{\pi}{a}} d^2k \frac{P_\mu(p) - P_\mu(p+k) - P_\mu(p-k) + P_\mu(p)}{K_\mu(k)K_\mu(k)} D_{\mu\mu}(k) \tag{34}$$

where $D_{\mu\mu}$ is the photon propagator. This expression is infrared singular for $p \sim \frac{\pi}{a}$. However it was shown in the preceeding section that the integration over p produces the factors $\sim k_\mu^2$ compensating this singularity. Therefore one can rescale the integration variables $k_\mu \to a^{-1}k_\mu$, which allows to estimate this integral when $a \to 0$:

$$\Pi \sim a \int_{-\pi}^{\pi} d^2x F(x, ap) \tag{35}$$

where it is understood that Π is attached to some fermion loop with the integration momentum p. According to the discussion above the integral is convergent and therefore $\Pi \to 0$ when $a \to 0$. Analogous reasonings show that all loops including internal photon lines vanish in the continuum limit. The only nonzero diagram is the photon polarization operator which coincides with the continuum result:

$$\Pi_{\mu\nu} = -\frac{e^2}{\pi}(\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}) \tag{36}$$

Note that if one would apply to our model Rabin procedure without additional PV regularization one would get for the mass gap the result which is the continuum one times $\sqrt{2}$ [7]. To avoid misanderstanding we emphasize that we did not solve exactly the Schwinger model for a finite lattice spacing, but rather demonstrated that the lattice approximation developed above generates in the continuum limit the correct exact solution of the Schwinger model if the limit is taken termwise in the weak coupling expansion.

4 Discussion

In this paper we showed that modification of the SLAC model by introducing the generalized PV regularization provides a manifestly gauge invariant lattice description of anomaly free chiral models. Our model does not suffer from fermion doubling and leads in the continuum limit to the usual gauge invariant results with only local counterterms. When applied to the anomaly free chiral Schwinger model it reproduces in the continuum limit the well known exact solution.

Aknowlegements.
I am grateful to J.Smit for helpful discussion.
This work was supported by International Science Foundation under grant MNB000 and Russian Basic Research Fund under grant 94-01-00300a.

References

[1] S.D.Drell, M.Weinstein and S.Yankielovitz, Phys.Rev.D14 (1976) 487,1627.

[2] Y.Aharonov, A.Casher and L.Susskind, Phys.Rev.D5 (1972) 1988.

[3] L.Susskind, Phys.Rev.D16 (1977) 3031.

[4] S.A.Frolov, A.A.Slavnov, Phys.Lett.B309 (1993) 344.

[5] L.H.Karsten, J.Smit, Nucl.Phys.B144 (1978) 536; L.H.Karsten, J.Smit, Phys.Lett.B85 (1979) 100.

[6] J.M.Rabin, Phys.Rev.D24 (1981) 3218.

[7] G.T.Bodwin, E.V.Kovacs, Phys.Rev.D35 (1987) 3198.

[8] K.G.Wilson, Phys.Rev. D10 (1974) 2445; *in New Phenomena in Subnuclear Physics.* A.Zichichi ed.Plenum N.Y. (1977).

[9] J.Smit, Nucl.Phys.B175 (1980) 307.

[10] P.D.V.Swift,Phys.Lett.B145 (1984) 256.

[11] J.Smit, Nucl.Phys.B (Proc. Suppl.)17 (1990) 3.

[12] M.F.L.Golterman, Nucl.Phys.B (Proc. Suppl.) 20 (1991) 528.

[13] D.N.Petcher, Nucl.Phys.B (Proc.Suppl.) 30 (1993) 50.

[14] R.Narayanan, H.Neuberger, Phys.Lett.B302 (1993) 62.

[15] S.A.Frolov, A.A.Slavnov, Nucl.Phys.B411 (1994) 647.

[16] A.A.Slavnov, Phys.Lett.B319 (1993) 231.